PHYSICS

B.Sc. Part (I) Physics (Hons.) I paper group(A) Spl. theory of Relativity

Expression for the Variation of mass with velocity for a relativistic particle :→

Let us consider two frame of references S and S'. Also let that in system S' two bodies of equal masses m' are travelling with velocities u' and −u' parallel to x-axis collides and after collision coalesce into one body. In relativity the principles of Conservation of mass and of momentum holds. On the basis of conservation of mass, the mass of the coalesced bodies after collision is equal to 2m' And on the basis of conservation of momentum coalesced bodies are at rest in S'. Let us now consider this impact appeared to an observer in S. The velocities u_1 and u' transform according to the law of addition of velocities with u_1 & u_2

$$u_1 = \frac{u' + v}{1 + \frac{u'v}{c^2}} \quad \text{and} \quad u_2 = \frac{-u' + v}{1 - \frac{u'v}{c^2}} \quad --- (1)$$

Let the mass of the body travelling with velocities u_1 and u_2 be m_1 and m_2 respectively. After collision the coalesced bodies travel with the velocity v w.r.to S since they are at rest in S'. As conservation of momentum holds for all frame of reference

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v \quad --- (2)$$

from eqns (1) & (2)

$$\frac{m_1}{m_2} = \frac{1 + \frac{u'v}{c^2}}{1 - \frac{u'v}{c^2}} \quad --- (3)$$

from eqn. (1) it can be prove that

$$\frac{u'v}{c^2} = \frac{2c^2 - u_1^2 - u_2^2 - 2\sqrt{(c^2 - u_1^2)(c^2 - u_2^2)}}{u_1^2 - u_2^2}$$

Putting this in eqn (3)

$$\frac{m_1}{m_2} = \frac{\sqrt{1-\frac{u_2^2}{c^2}}}{\sqrt{1-\frac{u_1^2}{c^2}}} \quad \text{------} (4)$$

If the body of mass m_2 is moving with zero velocity in S before collision so that $u_2 = 0$

$$m_1 = \frac{m_2}{\sqrt{1-\frac{u_1^2}{c^2}}} \quad \text{---} (5)$$

Now both bodies have the same mass when moving with the same velocity; so that the above equation means that m is the mass of the body when it is moving with velocity u_1 and m_2 when velocity is zero. Thus we may consider $m_1 = m$, $m_2 = m_0$, $u_1 = v$

So that $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{----} (6)$$

Hence masses of any moving body appear to increase with velocity becoming infinite when v attains the velocity of light c.

Q → Show that $E = mc^2$.
(mass & energy equivalence according to special theory of relativity):

Proof → As in classical dynamics, energy may be defined in terms of work, which is force × displacement and force being the rate of change of momentum, is given by

$$F = \frac{d}{dt}(mv)$$

But since both mass and velocity are variable in the theory of relativity

$$F = m\frac{dv}{dt} + v\cdot\frac{dm}{dt}$$

Suppose the force F acting through a distance dx raises the K.E. by dE_1 then

$$dE_1 = F\,dx$$

$$= m\frac{dv}{dt}\cdot dx + v\cdot\frac{dm}{dt}\cdot dx$$

$$\because v = \frac{dx}{dt}$$

$$dE_1 = mv\,dv + v^2 dm \quad \text{------} (1)$$

The relativistic law of variation of mass with velocity $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

can be written as by squaring both sides.

$$m^2 = \frac{m_0^2c^2}{c^2-v^2}$$

$$\text{or, } m^2c^2 - m^2v^2 = m_0^2c^2$$

on Differentiation

$$c^2\cdot 2m\,dm - v^2\cdot 2m\,dm - m^2\,2v\,dv = 0$$

$\because$ c and m_0 are constant while m and v are variables.

Dividing by $2m$ we get

$$c^2dm - v^2dm - mv\,dv = 0$$

$$\text{or, } mv\,dv + v^2dm = c^2\,dm$$

$$\therefore dE_1 = c^2\,dm \quad \text{-----} (2)$$

When the body is accelerated from zero velocity to v, its mass increases from m_o to m and the total K.E. is

$$\int dE_1 = E_2 = \int_{m_o}^{m} c^2 dm$$

$$= c^2 (m - m_o)$$

$$\therefore \quad E_2 = c^2 (m - m_o) \quad \text{---- (3)}$$

This relation indicates that the K.E. of motion is the actuating inflence of the increase of mass and rest mass m_o has to be understood as an internal store of energy in the body. Since the total energy E possessed by the moving body is made up of the K.E. of the motion and they stored up internal energy.

$$E = E_2 + m_o c^2$$

$$= (m - m_o) c^2 + m_o c^2$$

$$= mc^2 - \cancel{m_o c^2} + \cancel{m_o c^2}$$

or $\quad E = mc^2$

This is famous Einstein's mass energy relation, which states a universal equivalence between mass and energy.

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